Geometry, volume 1, Boston, 1910, p. 111). It is not known how Pascal proved the theorem, but probably his method was one of the methods used by Steiner.

4. Dean Boyd traced the development of theorems leading up to and supplementing Pascal's theorem, mentioning the work of Euclid, Pappus, Desargues and others. Most of the time was given to proofs of the theorems concerning the Pascal lines, the Steiner points, the Cayley-Salmon lines, the Kirkman points, the Steiner-Plücker lines, and the Salmon points. Carefully drawn figures illustrated the work.

5. Professor Richardson gave a brief introduction to the calculus of finite differences, and showed how this calculus when applied to Kramp's faculty function leads to simple methods for the summation of many series. The following types of Kramp's faculty were discussed and illustrated:

$$(a + bx)^{m/b}; \qquad \frac{1}{(a + bx)^{m/b}}; \qquad \frac{a^{x/b}}{c^{x/b}}.$$

Reference was made to Chrystal's *Algebra*, part II, chapter 31.

ELIZABETH LESTOURGEON, *Secretary-Treasurer.*

THE DECEMBER MEETING OF THE MARYLAND-VIRGINIA-DISTRICT OF COLUMBIA SECTION.

The tenth regular meeting of the Maryland-Virginia-District of Columbia Section of the association was held at Johns Hopkins University, Baltimore, Maryland, on December 10, 1921. Mr. O. S. Adams, chairman of the Section, presided at both morning and afternoon sessions. There were forty-four in attendance, including the following thirty members of the Association:

O. S. Adams, J. J. Arnaud, R. N. Ashmun, Clara Bacon, Sarah Beall, G. A. Bingley, C. C. Bramble, J. A. Bullard, G. R. Clements, A. Cohen, A. Dillingham, H. English, J. B. Eppes, H. H. Gaver, W. M. Hamilton, W. E. Heal, L. S. Hulburt, W. D. Lambert, A. E. Landry, Florence P. Lewis, E. S. Mayer, F. Morley, F. D. Murnaghan, J. R. Musselman, C. A. Nelson, C. H. Rawlins, H. M. Robert, Jr., R. E. Root, W. F. Shenton, C. A. Shook.

An amendment to the constitution of the Section was adopted, increasing the membership of the Executive Committee from three to four in order that Washington, Baltimore, Annapolis, and the state of Virginia might each have a representative on this committee.

The following resolution was introduced by Mr. W. D. Lambert, and adopted by the Section:[1]

[1] While the arrangement suggested by this resolution is clearly impractical as a working basis for the annual meetings of the Association, since all feeling of continuity or balance in the program would be sacrificed, there seems no reason why the section meetings should not respond as directly as possible to the several interests of the attendant members. The secretary of the Maryland-Virginia-District of Columbia Section refers to the Section as "nearly the banner section so far as attendance is concerned." In reference to this resolution, he remarks, "The

Resolved: (1) That the Section welcomes papers from any of its members, whether personally known to the officers of the Section or not, and that all members should feel free to present papers at the meetings.

(2) That it is intended to continue the present practice of holding meetings in May and December, and that any member desiring to present a paper should not wait for an invitation but should send an abstract several weeks in advance of the time for the meeting either to the officer of the Section representing that portion of the territory where the member lives, or directly to the Secretary-Treasurer, it being understood that the presentation of a paper may be postponed if the number of papers offered should be large, or if the Executive Committee has limited the topics for discussion at a particular meeting.

Professor A. Cohen reported for the Committee of the Section coöperating with the National Committee on Mathematical Requirements.

The value and pleasure of the meeting were enhanced by the fact that all those in attendance had lunch together, as guests of the members of the Department of Mathematics of Johns Hopkins University. The appreciation of the Section for the hospitality of these gentlemen throughout the day was expressed by a cordial vote of thanks.

The following papers were presented:

(1) "On a remarkable property of $(a + b\sqrt{-n})^n$, where n is of the form $4m + 3$" by Mr. WM. E. HEAL;

(2) "An arithmetical pyramid" by Dr. J. R. MUSSELMAN;

(3) "Multiple-valued solutions of Laplace's equations" by Mr. C. A. SHOOK;

(4) "Recent progress in hydrodynamics" by Professor F. D. MURNAGHAN;

(5) "An extension of Ptolemy's theorem" by Professor FRANK MORLEY;

(6) "Variation of latitude" by Mr. W. D. LAMBERT;

(7) "The fluctuating attitude toward mathematics" by Mr. HARRY ENGLISH;

(8) "Minimal curves" by Dr. L. M. KELLS, U. S. Naval Academy (Introduced by Professor G. R. Clements).

Abstracts of seven of the papers follow below, the numbers corresponding to the numbers in the list of titles:

1. Writing $(a + b\sqrt{-n})^n = P + Q\sqrt{-n}$, Mr. Heal inquired whether P and Q are each separable into three rational factors. It has been known since the time of Euler that this is true for $n = 3$. In fact $P = a(a + 3b)(a - 3b)$, $Q = 3b(a + b)(a - b)$. Mr. Heal has established the theorem for all prime values of n less than 100, and believes that it is true in general.

2. A three-dimensional series of numbers was built up by Dr. Musselman, following a given law, which cut by transverse planes form an arithmetical pyramid. The numbers in these planes can be used to give a solution of the Problem of Points for three players who are equally skillful.

3. In his paper Mr. Shook showed how to construct the Green's func-

general interest of the meetings could be increased, and the labors of the Executive Committee lightened, if the various members would feel more free to offer discussions on topics of either a scientific or pedagogic nature which they have found interesting.—EDITOR.

tion, G, for a Riemann space of two sheets. This is done as follows: Let R denote the distance from the fixed point (r', φ', z') to any point (r, φ, z). Let R' denote the value of R, when φ is replaced by the complex variable α. Let $f(\alpha)$ denote any analytic function of α which (a) has a simple pole with unit residue at the point $\alpha = \varphi'$ and (b) is periodic with period 4π in both α and φ'. Then form the integral with respect to α having for integrand $1/R' \cdot f(\alpha)$, the path being so chosen that it includes the point $\alpha = \varphi'$, but no other singularity of the integrand. This integral multiplied by $2\pi i$ is the desired function G, with pole at (r', φ', z'). This function is single-valued in the Riemann space, but in ordinary space it is double-valued. Let G' denote the Green's function with pole at $(r', -\varphi', z')$, namely the image of the pole of G in the branch membrane. Then the difference $G - G'$ turns out to be the potential due to a semi-infinite plane under the influence of a unit charge of electricity at the point (r', φ', z'). In this difference the value in the first sheet of space is the one taken. The pole of G', being in the second sheet, does not cause an infinity to occur at that point in the first sheet. In that position it is, so to speak, harmless. Several possible extensions were indicated, such as curvilinear branch line and closed branch line.

4. Dr. Murnaghan spoke on the recent advances in the theory of resistance of fluids which have been made in two different directions: (*a*) by the Italian and French mathematicians, following the lead of T. Levi-Civita, and H. Villat, and (*b*) by the German workers in applied mathematics, following the lead of Prandtl and von Karman. The results obtained by the Germans have been up to the present the more useful to the practical men who work on the design of aëroplanes.

6. Mr. Lambert's paper dealt with the latitude observations of the International Latitude Service and particularly with the observations at Ukiah, Calif. These show on their face a progressive increase of latitude there of almost 0".01 a year, an increase which has been interpreted by a well-known geologist as a northward creep of the surface crust at Ukiah over the underlying strata. Mr. Lambert showed that the other stations of the Latitude Service showed changes of latitude of the same order of magnitude, some having greater changes than Ukiah, and that all of these changes could be accounted for within a reasonable margin of error by assuming: (a) a purely fictitious change in latitude common to all stations and due to a cumulative error in the star places used; and (b) an actual change of latitude due to a displacement of the North Pole towards the American continent along the meridian of 77° West at the rate of about 0".0050 a year. These deductions are limited to the years 1900–1917 inclusive. It was found that the latitude observations were accurate enough to give some information regarding the figure of the earth. They confirm in a general way Helmert's results from gravity observations that if the Earth is considered as an ellipsoid of three unequal axes, the shortest equatorial radius is 230 meters less than the longest one and lies in longitude 107° West. Attention was also called to Problem 2872, proposed by the author in the MONTHLY for January, 1921;[1] the

[1] Solved in this number of the MONTHLY, 227.

results there stated were found useful in discussing the latitude observations. The detailed investigation will appear in print as a special publication of the U. S. Coast and Geodetic Survey (no. 80); a shorter account appeared in the *Journal of the Washington Academy of Sciences*, January 19, 1922.

7. Mr. English said that the general trend of all opinion in the world seems to be a fluctuating one. The attitude towards mathematics is no exception, as to importance of methods of study and content of courses. The western part of the United States seems now to be dominating the eastern part, though it is not any too sure of itself. The reorganization which it is zealously pushing has many good features, though seemingly only one year of high school mathematics is required and that is somewhat polyglot in its nature. It may well apply to a junior high school course if that is a school of completion, but if it is to be the universal preparation for all senior high school mathematics, there is real danger along the line through the high schools and up into the colleges. There is naturally a haziness about the upper years, which may result fatally, especially in the hands of the majority of teachers, as definite details must be given if no harm is to result to pupils going from one teacher to another. It is a case of making haste slowly, giving careful consideration to all factors involved.

8. Dr. L. M. Kells first reviewed the well-known theory for minimal curves in the plane and in space, and then showed that the general equations of minimal curves in 4-space are

$$x_1 = \tfrac{1}{2}[(\lambda^2 - 1)F_1''(\lambda) - 2\lambda F_1'(\lambda) + 2F_1(\lambda) + i\{(\lambda^2 + 1)F_2''(\lambda) - 2\lambda F_2'(\lambda) + 2F_2(\lambda)\}],$$

$$x_2 = -\tfrac{1}{2}i[(\lambda^2 + 1)F_1''(\lambda) - 2\lambda F_1'(\lambda) + 2F_1(\lambda) + i\{(\lambda^2 - 1)F_2''(\lambda) - 2\lambda F_2'(\lambda) + 2F_2(\lambda)\}],$$

$$x_3 = \lambda F_1''(\lambda) - F_1'(\lambda),$$

$$x_4 = \lambda F_2''(\lambda) - F_2'(\lambda),$$

where $F_1(\lambda)$ and $F_2(\lambda)$ are arbitrary functions of λ and the primes indicate derivatives with respect to λ. Properties in 4-space analogous to those for ordinary space were considered.

G. R. CLEMENTS, *Secretary-Treasurer.*

THE APRIL MEETING OF THE OHIO SECTION.

The seventh regular meeting of the Ohio Section was held in the Physics Building, Ohio State University, Columbus, on April 14–15, 1922, in connection with the meetings of the Ohio College Association and allied societies. An afternoon and an evening meeting were held on Friday. On Saturday a joint meeting of two sessions with the Ohio members of the Society for the Promotion of Engineering Education was held. Chairman B. F. Yanney presided, being relieved by Professor K. D. Swartzel for an interval.

There were fifty-three persons in attendance, including the following thirty-three members of the Association: